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THE QCD STRING WITH QUARKS.
I.SPINLESS QUARKS

Moscow - 1993

A b s t r a c t

Starting from the QCD Lagrangian we derive the effective action for massive quark and antiquark at large distances, corresponding to the minimal area low of the Wilson loop. The path integral method is used to quantize the system and the spectrum is obtained with asymptotically linear Regge trajectories. Two dynamical regimes distinguished by the string energy–momentum distribution are found: at large orbital excitations ($l \gg 1$) the system behaves as a string and yields the Regge slope of $\frac{1}{2\pi\sigma}$, while at small l one obtains a potential-like regime for relativistic or nonrelativistic system . The problem of relative time is clarified. It is shown that in the valence quark approximation one can reduce the initial four-dimensional dynamics to the three-dimensional one.

The limiting case of a pure string (without quark kinetic terms) is studied and the spectrum of the straight-line string is obtained.

1 Introduction

This is the first paper of the presumed series devoted to the quantum dynamics of the quark-antiquark system at large distances. Our starting point is the formalism of vacuum correlators, developed previously [1] (for a review see [2]). It allows one to represent the gauge-invariant Green's function of the quark-antiquark system in a form, where all dynamics is contained in the averaged Wilson loop operator.

We simplify our problem by disregarding effects due to the quark spins and additional quark loops (sea quarks) having in mind to come back to it in later papers.

In this way our simplified problem is that of a scalar quark and antiquark without additional quark loops in the confining background field.

One of the most important point of the paper is to identify an explicit mechanism creating the QCD string and to find the properties of the latter.

Even the question: what is it, the QCD string? is not trivial. Usually it is associated with the Nambu-Goto string action (for review see [3]) describing the open string with active degrees of freedom all along the string including the ends, where massless quarks are presumed to be described by the proper boundary conditions. This standard picture is plagued by unphysical features in 4 dimensions and needs $d = 26$ or supersymmetric extentions to be consistent [3].

The picture which emerges in our paper and based on the QCD Lagrangian and the vacuum correlator method essentially differs from the standard one. First, the string appearing in the $q\bar{q}$ system has a world-sheet coinciding with the surface appearing in the area law of the Wilson loop. This area law is a natural consequence of the cluster expansion for the Wilson loop [4], and the surface bounded by the Wilson contour appears also naturally in the formalism.

An important point is what kind of surface appears in the area law of the Wilson loop ? Since the whole cumulant series does not depend on the shape of the surface and has no degrees of freedom on it , we come to the conclusion that this should be the minimal surface which enters the area law and therefore defines also the shape of the string, connecting quark and antiquark. We stress however that this is an assumption we make here [1], and it is desirable to obtain

[1]This assumption is equivalent to the uniform convergence of the cluster series for the Wilson loop [4], since

the behaviour of the Wilson loop in QCD directly.

The QCD area law appears only asymptotically at large distances, $R \gg T_g$, where T_g is the so-called gluon correlation length, which enters as a scale for the vacuum correlators [5,1,2]. Recent Monte-Carlo calculations give the estimate $T_g \approx 0.2 \div 0.25 fm$ [6] and therefore we can apply our formalism even to the ground state mesons (see [2] for a discussion). We approximate the minimal area law surface as the world sheet surface of the straight line string connecting proper positions of quark and antiquark. We shall discuss in the next sections the validity of the straight-line approximation.

We consider the string tension as being renormalized and disregard all corrections including perturbative gluon exchanges and the creation of additional quark loops, since our aim here is to concentrate on the main dynamical ingredient – the interaction between quarks corresponding to the minimal area law.

Therefore in this approximation the minimal string may rotate (orbital excitations), and oscillate longitudinally (streching and expanding). The latter type of motion is not possible for the standard Nambu-Goto string.

After these introductory words about the definition of the QCD string, we can outline the purpose and the plan of the paper.

We shall quantize the system consisting of a massive or massless quark and antiquark connected by the minimal string.

We shall extensively use the method exploited recently by one of us [2,7] for this purpose. The method contains five steps. (1) First, one represents the quark-antiquark Green's function using the Feynman-Schwinger form [1] as a path integral over trajectories of q and $\bar{q}$ parametrized by their proper time parameters. (2) Second, all gluonic nonperturbative field contained in Wilson loops, is replaces by the minimal area law. (3) Third, the minimal area is assumed to be spanned by straight lines connecting a point on the q trajectory with another point on the $\bar{q}$ trajectory, these two points are chosen at the same proper time parameter. (4) The proper-time Hamiltonian is introduced to get rid of path integration and obtain instead differential equation. (5) Fifth, an approximation is made in the method [2,7] to expand the Nambu-Goto string term in the action in powers of some numerical parameter (which yields also expansion in powers of relative velocity $\dot{r}^2$).

Due to this expansion dynamics in relative time is free and one integrates it out, regaining in the end three dimensional (relativistic) dynamics with dy-

any term in the series provides the area law for large enough loop.

namically generated constituent mass.

This approximation [1,2,7] appeared to be simple and practical, yielding masses and Regge trajectories in terms of only one parameter – the string tension. In this way light mesons, heavy-light mesons [8], baryons [9] and glueballs [10] have been considered. The accuracy of the approximation for masses was estimated as $\sim 10\%$.

As a result [7] the slope of Regge trajectories was obtained to be $(8\sigma)^{-1}$ the same as in the potential models [11] in contrast to the Nambu–Goto string slope of $(2\pi\sigma)^{-1}$.

In the present paper we abandon this approximation and treat the effective action of the system exactly, using the formalism of auxiliary fields to get rid of the square root term [12,3]. As a result we shall not only improve the accuracy of the approximation but shall obtain a qualitatively different dynamics, which has not been present before in the approximation [2,7].

We shall find that there are two regimes of $q\bar{q}$ dynamics distingushed by the energy- momentum distribution of gluon fields, — for large orbital momenta l the resulting spectrum in the leading approximation coincides with that of a pure string with a slope $1/2\pi\sigma$, while for low values of l (depending on the masses of quarks) the dynamics is described by the relativistic potential-like approach,close to the one obtained previously [8,2]. For the heavy quark system the potential picture is valid in a large region of l and it joins smoothly the string picture for very large l.

The limiting case of pure strings (without kinetic terms of quarks) is considered in detail [2]. The straight-line Regge-trajectories, corresponding to the spectrum of the rotating straight-line string is obtained. This is in an agreement with the results obtained in [14]. Note that in our approach the string picture has not been assumed but it was derived from the QCD-Lagrangian under rather general assumptions.

The important issue of relativistic dynamical systems is the role of the relative time of q and $\bar{q}$. To make our straight-line approximation for the minimal surface selfconsistent we have to integrate over the class of paths without time backtracking of quarks (in the c.m.s.). This constraint corresponds to the valence quark approximation to the problem and allows one to rigorously reduce the initial four-dimensional (4D) dynamics to the three-dimensional (3D) one. As a result the quadratic 4D kinetic terms are transformed into 3D square-root-type terms. We have the Lorentz invariant effective action with the constraint,

[2] A short version of these results is reported in ref.[13]

which can be formulated in a Lorentz covariant form. So the calculation of the spectrum can be performed in an arbitrary frame and for the sake of convenience we work in the meson rest system.

The plan of the paper is the following. The Feynman-Schwinger representation (FSR) for the $q\bar{q}$ - Green's function is given and the effective action for $q\bar{q}$-system is obtained in Sec.2. In section 3 we obtain the approximate explicit expression for Wilson loop at large distances. In section 4 in the valence quarks approximation we integrate out time-components, so that we reduce the 4D dynamics to the three-dimensional one. Gaussian representation for the obtained action is formulated in Sec.5. The method of auxiliary fields [3],[12] is used in this section to get rid of the square root term, which determines the string action.

The case of a pure straight-line string without quarks at the ends is discussed in Section 6. In Section 7 Hamiltonian for the general case of the straight-line string with quarks at the ends is obtained and in two limiting cases $l \approx 1$ and $l \gg 1$ the analytic form of the spectrum is established.

In conclusions we summarize our main results and make comparison to those in literature.

Appendices A,B,C contain technical details of the derivation of formulas in the text.

2 Green's function of the quark-antiquark system interacting with gluon field.

In this Section we use the Feynman-Schwinger representation for the quark-antiquark Green's function to obtain the effective action for $q\bar{q}$ system in terms of the Wllson loop [1].

We start with the initial and final $q\bar{q}$ states defined on space-like surfaces in a gauge-invariant way

$$\Psi_{in}(y,\bar{y}) = \bar{u}(y)\Gamma_{in}(y,\bar{y})u(\bar{y}) \tag{1}$$

$$\Psi_{out}(x,\bar{x}) = \bar{u}(x)\Gamma_{out}(x,\bar{x})u(\bar{x}) \tag{2}$$

where $\Gamma_{in}, \Gamma_{out}$ contain a parallel transporter Φ and some vertex with definite Lorentz structure $\gamma_{in}, \gamma_{out}$:

$$\Gamma_{in} = \Phi(y,\bar{y})\gamma_{in}, \quad \Phi(y,\bar{y}) = Pexp(ig\int_{\bar{y}}^{y} A_\mu dz_\mu) \tag{3}$$

and the same for $\Gamma_{out}; \gamma_{in}$ may be e.g.γ_5, γ_μ etc. The operator P ensures the the ordering of $A_\mu(z)$ along the path $z_\mu(t)$. The Green's function $G(x\bar{x}/y\bar{y})$ is obtained by averaging the product $\Psi_{in}\Psi^+_{out}$ over all quark and gluonic fields inside the path integral with the usual QCD action

$$G(x\bar{x}/y\bar{y}) =< \Psi_{in}(y,\bar{y})\Psi^+_{out}(x,\bar{x}) >_{\Psi,A} \tag{4}$$

The quark degrees of freedom can be easily integrated out with the result (for a nonzero flavour channel)

$$G(x\bar{x}/y\bar{y}) =< tr\Gamma(\bar{x},x)S(x,y)\Gamma(y,\bar{y})S(\bar{y},\bar{x}) >_A \tag{5}$$

where $S(x,y)$ is the quark propagator in the gluonic field A_μ, and the averaging over gluonic fields now includes also the quark determinant (actually a product of determinants over all flavours):

$$< B >_A \equiv \int DAe^{-S(A)}B(A)\prod_i det(m_i + \hat{D}(A)) \tag{6}$$

For the quark propagator in the gluonic field A_μ one can use the Feynman-Schwinger representation [1,2,15]

$$\begin{aligned} S(x,y) &=< x \mid (m+\hat{D}(A))^{-1} \mid y >= \\ =&< x \mid (m-\hat{D}(A))\int_0^\infty dse^{-s(m^2-\hat{D}^2(A))} \mid y >= \\ &= (m-\hat{D}(A))_x \int_0^\infty dsDze^{-K}\Phi_\Sigma(x,y) \end{aligned} \tag{7}$$

where the following notations are used

$$K = m^2 s + \frac{1}{4}\int_0^s \dot{z}^2_\mu(t)dt, \qquad \dot{z}_\mu = \frac{dz_\mu(t)}{dt} \tag{8}$$

$$\Phi_\Sigma(x,y) = (P_\Sigma exp(g\int_0^s \Sigma_{\mu\nu}F_{\mu\nu}(z(t))dt))\Phi(x,y) \tag{9}$$

and $\Sigma_{\mu\nu} = \frac{i}{4}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$. The operators P_Σ ensure the proper ordered insertion of operators $\Sigma_{\mu\nu}F_{\mu\nu}$ along this path (for a discussion see [15]).

Insertion of (7) into (5) immediatly gives [8,15]

$$\begin{aligned} &G(x\bar{x} \mid y\bar{y}) = \\ \int_0^\infty dse^{-K}Dz\int_0^\infty d\bar{s}e^{-\bar{K}}D\bar{z} < &\gamma^+_{out}\Phi(\bar{x},x)(m-\hat{D}(A))_x\Phi_\Sigma(x,y)\times \\ &\times\gamma_{in}\Phi(y,\bar{y})(m-\hat{D}(A))_{\bar{y}}\Phi_\Sigma(\bar{y},\bar{x}) >_A \end{aligned} \tag{10}$$

Analysis of the factors $(m - \hat{D})$ made in [9] shows that it can be replaced by $(m + \frac{1}{2}\hat{\dot{z}})$ and taken out of the angular brackets, so that we are left with the only term defining all the dynamics of the system, both perturbative and nonperturbative, which we call W_Σ.

The presence of spin degrees of freedom in (7) makes dynamics rather complicated. For this reason we shall omit the spin dependence and concentrate on the confining dynamics without spins. We also neglect the additional quark loops, which corresponds to the substitution of Πdet in eq. (6) by 1. This is in line with usual quenched approximation and can be justified in the $1/N_c$ expansion.

The resulting Green's function in this case has the form

$$G(x\bar{x} \mid y, \bar{y}) = \int_0^\infty ds \int_0^\infty d\bar{s} e^{-K-\bar{K}} Dz D\bar{z} < W(C) >_A \tag{11}$$

where $W(C)$ is the usual Wilson loop operator

$$W_\Sigma(C) = tr\Phi(\bar{x}, x)\Phi_\Sigma(x, y)\Phi(y, \bar{y})\Phi_\Sigma(\bar{y}, \bar{x}) = trPexp[ig \int_C A_\mu dz_\mu] \tag{12}$$

The closed contour C consists of initial and final pieces $[x, \bar{x}], [y, \bar{y}]$ and paths $z(t), \bar{z}(\bar{t})$ of the quark and antiquark.

It is convenient introduce as in [2] the following "center of mass" coordinate R_μ and the "relative" coordinate r_μ

$$R_\mu = \frac{s\bar{s}}{s + \bar{s}}(\frac{1}{s}z(t) + \frac{1}{s}\bar{z}(t))$$

$$r_\mu = z(t) - \bar{z}(t)$$

with the boundary condition imposed in the Lorentz covariant way

$$R_\mu(s) - R_\mu(0) = Tu_\mu \quad u^\mu u_\mu = 1$$

and $r_\mu(s) = r_\mu(0)$ are also fixed.

In what follows we shall be interested in the limit $T \to \infty$ to consider asymptotical states of the system. In this limit our boundary condition corresponds to the total momentum of the meson $P_\mu \sim u_\mu$.

3 Evaluation of the Wilson loop at large distances

In this section we shall obtain approximate expression for $< W(C) >_A$ at large distances.

Using the cluster expansion we can represent $< W(C) >$) as [4]

$$< W(C) >= exp \sum_{k=1}^{\infty} \frac{(ig)^k}{k!} \int d\sigma(1)...d\sigma(k) \ll F(1)...F(k) \gg \qquad (13)$$

where cumulants are irreducible averages [16,17] and we omit Lorenz indices in $d\sigma_{\mu\nu}(u(i))$ and $F_{\mu\nu}(u(i))$.

As shown in [2,18], there are three possible regimes for $< W(C) >$ depending on the relation between the sizes of the loop C and the correlation length T_g , defining the decay of cumulants $\ll F(1)...F(i)...F(j)...F(k) \gg \sim exp(-\frac{|z(i)-z(j)|}{T_g})$.

If we represent the loop C as a rectangular of time length T and space width R, then we have (omitting perimeter-type terms always coming from the quark-mass renormalization and exchanges) [2] '

$$i) < W(C) >\approx exp(-\sigma S), R \gg T_g, T \gg T_g \qquad (14)$$

$$ii) < W(C) >\approx exp(-T(c_2R^2 + c_4R^4 + ...)), R \ll T_g, T \gg T_g \qquad (15)$$

$$iii) < W(C) >\approx exp(-\frac{S^2g^2 < F^2(0) >}{24N_c} + 0(S^3)), R \ll T_g, T < T_g \qquad (16)$$

Here S is the area of the surface inside the contour C, which we take as the minimal surface, since $< W(C) >$ does not depend on the shape of the surface [17].

Thus we see that the area law (14) and the ensuing string dynamics appears only as the asymptotic regime for large contours, while for small or narrow loops the area law (and the QCD string) is absent and is replaced by a much weaker interaction.

In what follows we concentrate on the regime (14), i.e. we shall consider only large loops, $R, T \gg T_g$.

We note that in the Monte Carlo calculations this law has been seen in the wide region of $R, 0.1 \preceq R \preceq 1fm$ [19] also in the presence of dynamical quarks (the determinantal term in (6)) [20]. From those results one could conclude that T_g is small enough, so that the regime (14) is dominant for light quark systems.

Independent Monte-Carlo calculations [6] have confirmed that $T_g \cong 0.2 \div 0.3fm$, which should be compared with characteristic quark relative distance and orbiting time $R \sim T_q \approx 1fm$ for light quark system [2]. Thus we can conclude that the area law (14) is a reasonable approximation for light quark systems (even for ground states) and for excited states of heavy quarkonia.

The "minimal area law" implies the neglect of gluonic excitations above the QCD vacuum, which could lead to an effective integration over the surfaces, bounded by the contour C.

Next step is to construct explicitly the minimal area S in terms of a given contour C, defined by quark and antiquark paths $z_\mu(t)$ and $\bar{z_\mu}(t)$.

Any surface can be parametrized by the Nambu-Goto form

$$S = \int_0^T d\tau \int_0^1 d\beta [\dot{w}_\mu^2 w_\mu'^2 - (\dot{w}_\mu w_\mu')^2]^{1/2} \tag{17}$$

where $w_\mu(\tau, \beta)$ are the coordinates of the string world surface, and

$$\dot{w}_\mu = \frac{\partial w_\mu}{\partial \tau}, w_\mu' = \frac{\partial w_\mu}{\partial \beta}$$

.

We use the approximation that the minimal surface for given paths $z_\mu(\tau)$, $\bar{z}_\mu(\tau)$ is determined by eq.(17) with w_μ given by straight lines, connecting points $z_\mu(\tau)$ and $\bar{z}_\mu(\tau)$ with the same τ, i.e.

$$w_\mu(\tau, \beta) = z_\mu(\tau) \cdot \beta + \bar{z}_\mu(\tau)(1-\beta) \quad , \quad 0 \preceq \beta \preceq 1 \tag{18}$$

where τ is defined for both trajectories as

$$\tau = \frac{t}{s} T = \frac{\bar{t}}{\bar{s}} T \tag{19}$$

and T enters boundary condition. We have no proof that eq. (18) indeed fits the minimal area S in all cases; we note that this approximation is valid in two limiting cases which are of special interest below: in the case $l = 0$ one can exploit the flat dynamics of quarks and in the limit $l \to \infty$ corresponding to the dynamics of the string with typical trajectories of the double helycoid type, for which the minimal area indeed is formed by the straight-lines. In what follows we shall use eqs. (14,17,18) to express $< W(C) >$ in terms of quark coordinates.

We introduce a compact notation

$$\xi \equiv \{\tau, \beta\} \quad , \quad g_{ab}(\xi) \equiv \partial_a w_\mu \partial_b w^\mu, \quad a, b = \tau, \beta \tag{20}$$

and obtain

$$S = \int d^2\xi \sqrt{detg}. \tag{21}$$

4 The reduction of four-dimensional dynamics to the three-dimensional one

Before the insertion of the concrete form of $< W(C) >$ let us consider the integration over $z_0(t), \bar{z}_0(t)$ and its physical interpretation (in the meson rest system). These integrations (additional with respect to the nonrelativistic case) are related to the following phenomena. First, the relativistic quantum theory in general is the theory with unconserved number of particles. Even in the case of a free particle the backtracking of the time-component which corresponds to the creation of additional pairs at the intermediate stage leads to the proper relativistic structure of Green's function. Second the fact, that the interaction time is smeared out, generates a need for the integration over the relative time of particles in the process even in the valence quark approximation.

We suggest the following approach to the problem. At first step we separate out from the initial sum over all paths the class of the trajectories without backtracking of time-components, satisfying the condition (in the rest system)

$$\frac{dz_0(\gamma_1)}{d\gamma_1} > 0 \ , \quad \frac{d\bar{z}_0(\gamma_2)}{d\gamma_2} > 0 \tag{22}$$

or rewriting it in the explicitlly Lorentz-invariant form

$$(P^\mu \; dz_\mu(\gamma_1)/d\gamma_1) > 0 \ , \quad (P^\mu d\bar{z}_\mu(\gamma_2)/d\gamma_2) > 0 \tag{23}$$

where P_μ is the total 4-momentum of the meson.

This constraint has to be imposed, because for the trajectories with such backtracking the minimal surface which appears in $< W(C) >$ obviously can not be properly approximated by our straight line anzatz. This procedure corresponds to the usual valence quarks approximation and in the language of the time ordered diagrams the trajectories with backtracking corresponds to the production of $q\bar{q}$ - mesonic states.

In Appendix A it is shown, that for the trajectories without time-backtracking the variables $z_0(\gamma_1)\ , \quad \bar{z}_0(\gamma_2)$ can be transformed into the new nondynamical ones $\mu_1(\tau)\ , \quad \mu_2(\tau)$ without derivative terms by using the parametrization for which

$$z_\mu = (\tau \ , \vec{z}) \quad , \bar{z}_\mu = (\tau \ , \ \vec{\bar{z}}) \tag{24}$$

The integration over these new variables for the spinless quarks is performed effectively by steepest decent method and leads only to the modification of the

initial kinetic terms $K, \bar{K}$ (8) in the quark Green's function in an external field (7). The final action becomes independent of the time-components of $z_\mu, \bar{z}_\mu$.

$$G(x\bar{x}/y\bar{y}) = \int D\mu_1\ D\mu_2\ D\vec{z}\ D\vec{\bar{z}}\ exp[-K' - \bar{K}'] < W(C) >_A \tag{25}$$

where

$$K' + \bar{K}' = \int\limits_0^T \frac{d\tau}{2}[(\frac{m_1^2}{\mu_1(\tau)} + \mu_1(\tau)\{1 + \dot{\vec{z}}^2(\tau)\}) + (\frac{m_2^2}{\mu_2(\tau)} + \mu_2(\tau)\{1 + \dot{\vec{\bar{z}}}^2(\tau)\})] \tag{26}$$

We have introduced the new proper time parameter τ , $0 \leq \tau \leq T$, and

$$\mu_1(\tau) = \frac{T}{2s}\dot{z}_0(\tau)\ , \quad \mu_2(\tau) = \frac{T}{2\bar{s}}\dot{\bar{z}}_0(\tau) \tag{27}$$

with the dot standing for the derivative over τ. It should be stressed that the substitution of eq.(24) into $< W(C) >$ is implied. This condition is equivalent to

$$R_0(\tau) = \tau, \quad r_0(\tau) = 0 \tag{28}$$

which corresponds to the usual instant plane Hamiltonian dynamics in the c.m. system. We also observe in (28) that τ plays the role of the time for the meson – a common time for both quarks. We shall prove, that in the case of the pure straight-line string the condition

$$(Pr) = 0$$

directly follows from the dynamics of the string. This result supports the validity of our approximation of neglecting (in the straight-line approximation for the minimal surface) of backward in time trajectories in the rest system. It is important to stress, that this assumption can be not valid for the calculation of quantities other than the mass spectrum and for interactions different from the confining string-like interaction of quarks, which we have formulated.

Therefore we have shown that the starting 4D dynamics can be reduced to the 3D one. And it should be emphasized that in general this is a nonlocal 3-dimensional dynamics due to the double independent integration in $W(C)$ over $\vec{z}(\tau)$ and $\vec{\bar{z}}(\tau)$, which bindes together $\vec{z}(\tau)$ and $\vec{\bar{z}}(\tau)$ with different arguments τ_1 and τ_2.

In addition we have two additional integrations over $D\mu_1$, and $D\mu_2$ which replace integrations over time components $z_0, \bar{z}_0$, and one common evolution parameter in the action which can be identified with the c.m. time. The

physical meaning of μ_1, μ_2 can be most easily clarified imposing an external e.m. field, e.g. in calculating the magnetic moment. Then it can be shown, that μ_i enter magnetic moment as the constituent mass of a quark. The same happens with spin-dependent forces (to be published). Therefore we shall call μ_i the dynamical mass of the quark i. In the approximation when μ does not depend on τ, this dynamical mass has been introduced in [2,7]. As we shall see below this approximation works reasonably well – accuracy in the determination of mass is around 5% (see e.g. Table 4 of [2]).

5 Gaussian representation for the effective action of quarks and the string.

Combining the results of the previous section we obtain the total effective action.

$$A \equiv K' + \bar{K}' + \sigma_0 \int_0^T d\tau \int_0^1 d\beta \sqrt{detg} \tag{29}$$

This action is similar to the one considered in the papers [15] , where the straight-line string without transverse excitations has been considered. However in our case also kinetic terms of quarks $K' + \bar{K}'$ are present and the condition that the ends off the string move with the velocity of light is not possible.

A direct procedure of quantization of (29) is difficult due to the square root term and we use the auxiliary fields approach [3] to get rid of it. In the Appendix B we give a detailed derivation of this procedure , while here we present the final gaussian representation of this action

$$\begin{aligned} A &= \int_0^T d\tau \int_0^1 d\beta \{ \frac{1}{2}(\frac{m_1^2}{\mu_1(\tau)} + \frac{m_2^2}{\mu_2(\tau)}) + \frac{1}{2}\mu_+(\tau)\dot{R}^2 + \frac{1}{2}\tilde{\mu}(\tau)\dot{r}^2 + \\ &+ \frac{1}{2\tilde{\nu}}[\dot{w}^2 + (\sigma\tilde{\nu})^2 r^2 - 2\eta(\dot{w}r) + \eta^2 r^2] \} \end{aligned} \tag{30}$$

where

$$\mu_+(\tau) = \mu_1(\tau) + \mu_2(\tau) \ , \quad \tilde{\mu}(\tau) = \frac{\mu_1(\tau) \cdot \mu_2(\tau)}{\mu_1(\tau) + \mu_2(\tau)}$$

and the condition (24) for $z_\mu(\tau), \bar{z}_\mu(\tau)$ is assumed. Since only extremal values of $\mu_1(\tau)$ and $\mu_2(\tau)$ contributes one can intgrate in our symmetric case over restricted class of functions $\mu_1(\tau) = \mu_2(\tau) = \mu(\tau)$. Here $\tilde{\nu}(\tau, \beta) \geq 0$ and $\eta(\tau, \beta)$

are two auxiliary fields, which should be integrated out in the full path integral representation for G:

$$G = \int DRDrD\tilde{\nu}D\eta D\mu e^{-A} \tag{31}$$

In order to prove the equivalence of the dynamics governed by the actions (29) and (30) it is enough to show that after proper integration over $\tilde{\nu}(\tau,\beta), \eta(\tau,\beta)$ one returns from eq. (30) back to the initial one (29).

Actually after the gaussian integration over $\eta(\tau,\beta)$ we obtain instead of eq. (30) the following expression for G

$$G = \int DRDrD\mu exp[-K' - \bar{K}']D\tilde{\nu}\times \tag{32}$$

$$exp[-\int_0^T d\tau \int_0^1 d\beta \frac{1}{2}[\frac{\dot{w}^2 w'^2 - (\dot{w}w')^2}{(r^2\tilde{\nu})} + \sigma^2(\tilde{\nu}r^2)]]$$

where we restore the notation

$$w'_\mu = r_\mu$$

.

For the integral over $\tilde{\nu}$, as it is shown in Appendix A, the explicit measure can be constructed and the effective action is determined only by the extremum value of $\tilde{\nu}$.

$$\sigma^2 r^4 \tilde{\nu}_0^2 = (\dot{w}^2 w'^2 - (\dot{w}w')^2) \tag{33}$$

Inserting this expression for $\tilde{\nu}$ into the eq.(32) one recovers our starting action (29).

We emphasize that the integration over $\tilde{\nu}$ and η effectively amounts to the replacement of them by their extremum values.

The resulting action (30) is quadratic in R_μ, r_μ and can be conveniently rewritten as (here $\mu_1 = \mu_2 = \mu, m_1 = m_2 = m$)

$$A = \int_0^T d\tau[\frac{m^2}{\mu} + \frac{1}{2}\{a_1\dot{R}^2 + 2a_2(\dot{R}\dot{r}) - 2c_1(\dot{R}r) - 2c_2(\dot{r}r) + a_3\dot{r}^2 + a_4 r^2\}] \tag{34}$$

where we have used $w_\mu = R_\mu + (\beta - 1/2)r_\mu$ to express $\dot{w}$ in terms of $\dot{R}, \dot{r}$. We have introduced the following notations

$$\begin{gathered} a_1 = \int_0^1 d\beta(2\mu + \nu), \quad a_3 = \int_0^1 d\beta(\frac{\mu}{2} + (\beta - \frac{1}{2})^2\nu) \\ a_2 = \int_0^1 d\beta(\beta - \frac{1}{2})\nu, \quad a_4 = \int_0^1 d\beta(\frac{\sigma^2}{\nu} + \eta^2\nu) \\ c_1 = \int_0^1 d\beta\eta\nu, \quad c_2 = \int_0^1 \eta\nu(\beta - \frac{1}{2})d\beta \end{gathered} \tag{35}$$

and

$$\nu(\tau,\beta)=\frac{1}{\tilde{\nu}(\tau,\beta)}$$

plays the role of the dynamical energy density for the string. Since only $\dot{R}_\mu$ enters the action then it is convenient to integrate over $D\dot{R}$ taking into account boundary conditions $R_\mu(T)-R_\mu(0)=u_\mu T$, i.e.

$$\int DR\to\int D\dot{R}\int_{-i\infty}^{i\infty}d^4\lambda exp(\lambda_\mu\int_0^T(\dot{R}_\mu-u_\mu)d\tau) \tag{36}$$

We shall systematically disregard in what follows the preexponential factors and shall be interested only in the effective action in the exponent. In this way integrating out $D\dot{R}$ as in (36) we obtain

$$G\sim\int DrD\nu D\eta D\mu d^4\lambda\exp(-\tilde{A}) \tag{37}$$

where

$$\tilde{A}=\frac{1}{2}\int_0^T d\tau\{2(\lambda u)+\frac{2m^2}{\mu}+\frac{1}{a_1}[(a_3a_1-a_2^2)\dot{r}^2+2(c_1a_2-c_2a_1)(r\dot{r})$$

$$+(a_4a_1-c_1^2)r^2+2a_2(\lambda\dot{r})-2c_1(\lambda r)-\lambda^2\} \tag{38}$$

For generality we preserve the explicitly Lorentz- invariant form of the action, with the conditions $R_0=1, r_0=0$ being implied. It apparently amounts to the replacement in eq.(36) of an ordinary integration $d\lambda_0$ by the functional one $D\lambda_0(\tau)$. The expression (38) for the action $\tilde{A}$ formes the basis of our further calculations.

6 A quantization of the pure straight-line string

In this section the case of the pure straight-line string will be studied. It appeares that in this approximation to the full theory one doesn't need to use the conditions $(Pr)=0$. We shall show that in this case the reparametrization symmetry (B.3) dynamically induces this condition. We shall also obtain in this section the spectrum of this particular system.

In Appendix B it is shown that for the straight line string without kinetic terms of quarks at the ends the effective action (38) can be reduced to the following one

$$G=\int\ DR_\mu\ Dr_\mu\ D\eta(\tau,\beta)\ D\tilde{\nu}(\beta) \tag{39}$$

$$exp[-\int\limits_0^T d\tau\int\limits_0^1 d\beta\frac{1}{2\tilde{\nu}}[\dot{w}^2+((\sigma\tilde{\nu})^2+\eta^2)r^2-2\eta(\dot{w}r)]]$$

with $\tilde{\nu}(\beta)$ being independent on τ.

As well as in the previous section it is convenient to introduce instead of $\tilde{\nu}(\beta)$ a new variable,

$$\nu(\beta)=1/\tilde{\nu}(\beta) \tag{40}$$

which plays the role of an effective energy density of the string.

Let us first consider an integration over $D\eta(\tau,\beta)$. The functions $\eta(\tau,\beta)$ enter the action as an integral over β with functions $\nu(\beta)$. Only the extremal values of the function $\nu(\beta)$, which are even under the exchange $\beta-\frac{1}{2}\to-(\beta-\frac{1}{2})$ (due to the symmetry under the permutation of the ends of the string) contributes to the action. So we shall integrate only over the class of functions $\nu(\zeta)(\zeta=\beta-\frac{1}{2})$, which are even functions of ζ. It is convenient to decompose the functions $\eta(\tau,\beta)$ into orthogonal polinomials $P_n(\beta)$ with the weight $\nu(\beta)$

$$\eta(\tau,\beta)=\sum_n P_n(\beta)k_n(\tau) \tag{41}$$

$$\int_0^1 d\beta\nu(\beta)P_n(\beta)P_m(\beta)=\delta_{mn} \tag{42}$$

Taking into account the symmetry of $\nu(\beta)$, one can easily obtain expressions for $P_0(\beta)$

$$\begin{gathered}P_0(\beta)=N_0,\quad N_0^2=(\int_0^1\nu(\beta)d\beta)^{-1}\\ P_1(\beta)=(\beta-1/2)N_1,\quad N_1^2=(\int_0^1\nu(\beta)(\beta-1/2)^2d\beta)^{-1}\end{gathered} \tag{43}$$

The action (39) can be written in terms of N_i in the following form

$$\begin{aligned}S&=\frac{1}{2}\int_0^T d\tau[\frac{1}{N_1}\dot{r}^2+\sigma^2\int\frac{d\beta}{\nu(\beta)}r^2+r^2\sum_{n=1}^\infty k_n^2(\tau)-2N_0k_0(\tau)i(\lambda r)-\\ &-\frac{2}{N_1}(\dot{r}r)k_1(\tau)+\lambda^2N_0^2+2i(\lambda u)]\end{aligned} \tag{44}$$

with λ being replaced by $i\lambda$.

The function $k_0(\tau)$ enters only in the 4th term in this expression and an integration over $Dk_0(\tau) = \prod_{i=1}^{N} dk_0(\tau_i)(N \to \infty)$ gives the factor proportional to a product of δ-functions;

$$\prod_{i=1}^{\infty} \delta(\lambda r(\tau_i)) \tag{45}$$

Thus only the components of r_μ transverse to the direction of λ_μ, which plays the role of the total momentum P_μ should be taken into account and we have

$$\int D^4 r \delta(\lambda r) exp[-S] \to \int D^3 r\ exp[-S] \tag{46}$$

After the integration (46) the dependence of S on λ has a simple form

$$T(\frac{\lambda^2}{a_1} + 2i(\lambda u)) \tag{47}$$

and the integral over λ is saturated in the case $T \to \infty$ we are interested in by an extremum value

$$\lambda_\mu = i\frac{1}{N_0^2} u_\mu = P_\mu \tag{48}$$

Integration over Dk_n with $n \geq 1$ leads effectively to the following expression

$$\int d\nu(\beta) D^4 r \delta(ru) exp[-\frac{1}{2}\int_0^T d\tau[\frac{1}{N_0^2} + (\dot{r}^2 - \frac{(r\dot{r})}{r^2})\frac{1}{N_1^2} + \sigma^2 \int \frac{d\beta}{\nu(\beta)} r^2] \tag{49}$$

It is important that we have obtained the following constraints

$$(ru) \sim (rP) = 0 \tag{50}$$

$$(rp) = 0 \tag{51}$$

where P_μ is the total momentum of the string and

$$p_\mu = (\dot{r}_\mu - \frac{(r\dot{r})}{r^2} r_\mu)\frac{1}{N_1^2} \tag{52}$$

is the relative momentum of the string.

The second constraint (51) means that only transverse components of p enter the action. The same constraints appear in the canonical quantization of the straight-line string [14] .

Consider the rest system of the meson $u_\mu = (1, \vec{0})$ and transform the expressions from the Euclidean to the Minkowski space

$$d\tau_E \to i d\tau_M \tag{53}$$

It follows from eq. (49) that the hamiltonian of the problem is

$$H(\vec{p},\vec{r}) = \frac{1}{2}\{N_1^2\frac{\hat{L}^2}{\vec{r}^2} + \sigma^2\int\frac{d\beta}{\nu}\vec{r}^2 + \frac{1}{N_0^2}\} \tag{54}$$

where $\hat{\vec{L}} = (\vec{r}\times\vec{p})$ is the operator of the angular momentum.

This hamiltonian does not contain the radial part of the kinetic term so that the field $\vec{r}^2$ is not a dynamical one. Noticing that the integral over $\vec{r}^2$ has the form (A.13), one concludes that in the effective action only the extremum of $\vec{r}^2$ contributes. So that after solving the equation of motion (for a fixed value of orbital momentum)

$$-\frac{l(l+1)}{\vec{r}^4}N_1^2 + \sigma^2\int\frac{d\beta}{\nu} = 0 \tag{55}$$

we arrive at the final expression for the hamiltonian as follows

$$H(\nu,l) = \frac{1}{2}\frac{1}{N_0^2} + \sigma\sqrt{\int\frac{d\beta}{\nu}N_1^2}\sqrt{l(l+1)} \tag{56}$$

The function $\nu(\beta)$ has the form which gives the minimum of this hamiltonian (for details see ref.[13])

$$\nu_l(\beta) = (\frac{8\sqrt{l(l+1)}\sigma}{\pi})^{1/2}\frac{1}{\sqrt{1-4(\beta-\frac{1}{2})^2}} \tag{57}$$

This solution corresponds to the spectrum of the hamiltonian

$$E_l^2 = M_l^2 = 2\pi\sigma\sqrt{l(l+1)} \tag{58}$$

which agrees with the result obtained for the straight-line string in the canonical formalism [14].

Expression (57) could be physically interpreted if one notices that a parameter

$$v(\beta) = 2(\beta - 1/2) \tag{59}$$

plays the role of the velocity of the corresponding elementary piece of the string. In this way formula (57) can be rewritten into more familiar one

$$\nu_l(\beta) = \frac{\rho_l}{\sqrt{1-v^2(\beta)}} \tag{60}$$

where $\rho_l = (\frac{8\sigma\sqrt{l(l+1)}}{\pi})^{1/2}$ corresponds to the effective mass density of the string in the rest frame.

To conclude this section let us give the physical interpretation of the constraint (50), which shows that (in the rest system) the relative time is unimportant for the dynamics of the pure straight-line string. The given realization of the string doesn't include internal interaction between neighbouring points of the string, so that there is no physical exchange along the string. Because of this fact there is no need for the introduction of relative time dynamics.

7 The general case of the QCD string with quarks

This is the central part of our paper. In this section we shall derive the effective hamiltonian for the general case of the straight-line QCD-string with quarks and shall find the spectrum of the problem. As it has been done in the previous section, we integrate over $\eta(\tau,\beta)$, expanding it in orthogonal polinomials $P_n(\beta)$ with weight $\nu(\tau,\beta)$ After this integration and integration over $\vec{\lambda}$ in the c.m. system $\vec{u}=0$ we obtain for the equal mass case $(\mu_1(\tau)=\mu_2(\tau)=\mu((\tau))$

$$S=\int_0^T d\tau(\frac{m^2}{\mu(\tau)}+\mu(\tau)+\frac{1}{2}\{\frac{\mu(\tau)\dot{\vec{r}}^2}{2}+ \tag{61}$$

$$\int(\beta-1/2)^2\nu d\beta\frac{(\dot{\vec{r}}\times\vec{r})^2}{\vec{r}^2}+\int\frac{\sigma^2 d\beta}{\nu}\vec{r}^2+\int\nu d\beta\}$$

which will form the basis of our further discussions.

It should be stressed, that the absence of the dependence on the relative time $r_0(\tau)$ (the so-called instantaneous interaction) doesn't mean of course that this effective action is induced by the interaction between quarks and string during an infinitely small time interval.

Actually, we have managed to tranform the integration over $r_0(\tau)$ into the one over $\mu(\tau)$. And it is the peculiar property of the considered interaction that after this transformation one can (under the approximation discussed above) arrive at a local three-dimensional dynamics.

We shall now consider first the case of heavy masses and then arbitrary masses with l increasing from $l=0$ to $l=\infty$.

The limit of nonrelativistic potential dynamics.

This is the case when $m \gg \sqrt{\sigma}$, so that

$$m \sim <\mu> \gg <\nu> \tag{62}$$

An extremal equation for $\mu(\tau)$

$$\frac{m^2}{\mu^2(\tau)} = 1 + \frac{\dot{\vec{r}}^2}{4} \tag{63}$$

gives after taking into account a nonrelativistic condition

$$\dot{\vec{r}}^2 \ll 1 \tag{64}$$

the simple solution in the leading order

$$\mu(\tau) = m \tag{65}$$

From the extremal condition for $\nu(\tau, \beta)$

$$\frac{\sigma^2}{\nu^2(\tau\ ,\ \beta)}\vec{r}^2 = 1 + (\beta - 1/2)^2 \frac{(\dot{\vec{r}} \times \vec{r})^2}{\vec{r}^2} \tag{66}$$

one obtains in the leading order

$$\nu(\beta\ ,\ \tau) = \sigma\sqrt{\vec{r}^2} \tag{67}$$

and finally the action in the same approximation is

$$S = \int_0^T d\tau[2m + \frac{m}{4}\dot{\vec{r}}^2 + \sigma\sqrt{\vec{r}^2}] \tag{68}$$

as one can expect from the very beginning.

The general case. The transition from the potential dynamics to the string dynamics.

We start with the case $l = 0$ and arbitrary masses m

$$L^2 \sim (\dot{\vec{r}} \times \vec{r})^2 = 0 \tag{69}$$

One obtains from (61) with the help of (66)

$$S = \int_0^T d\tau[\frac{m^2}{\mu(\tau)} + \mu(\tau) + \frac{\mu(\tau)}{4}\dot{\vec{r}}^2 + \sigma \mid \vec{r} \mid] \tag{70}$$

In the Minkowski space-time the action (70) yields the Hamiltonian

$$H=\mu(\tau)+\frac{\vec{p}^2+m^2}{\mu(\tau)}+\sigma\mid\vec{r}\mid \tag{71}$$

and the extremal condition for $\mu(\tau)$ is

$$\mu(\tau)=\sqrt{\vec{p}^2+m^2} \tag{72}$$

Therefore we arrive at the following Hamiltonian in the case $l=0$

$$H(\vec{p},\vec{r})=2\sqrt{\vec{p}^2+m^2}+\sigma\mid\vec{r}\mid \tag{73}$$

where $\vec{p}^2=(\vec{p}\vec{r})^2/\vec{r}^2\equiv p_r^2$

This expression is widely used in the context of the so-called "relativistic quark model" [11]. As was discussed in [2,7] the eigenvalues of (73) differ only a little from the approximate version of this Hamiltonian used by one of the authors. There eq.(71) has been used with μ independent of τ. As a consequence, the procedure was to find first eigenvalues of (71) $E(\mu)$ as a function of μ, and then to minimize $E(\mu)$ with respect to μ, i.e. to find $\mu=\mu_0$ from $\frac{dE}{d\mu}=0$, and to calculate $E(\mu=\mu_0)$. As can be seen from Table 4 of [2], the eigenvalues ε_n of (73) and $E_n(\mu_0)$ differ at most by 5% for lowest states, while calculations of $E(\mu_0)$ are much easier to do, especially for many-quark and gluon states.

Let us discuss how the potential $\sigma\mid\vec{r}\mid$ obtained in the rest frame is transformed under Lorentz boosts. One should keep in mind that it is induced by the area law of Willson's loop, which is a Lorentz scalar. Therefore it is not difficult to verify that this potential is a Lorentz scalar also and in arbitrary frame it can be represented as

$$\sigma(r_\mu^2-\frac{(Pr)^2}{P^2})^{1/2}$$

where P is the total 4-momentum of the hadron .

In the case of small values of l, the string contribution to the kinetic part of the action (and to the total orbital momentum)

$$\int(\beta-1/2)^2\nu(\beta)d\beta\frac{(\dot{\vec{r}}\times\vec{r})^2}{\vec{r}^2} \tag{74}$$

can be treated as a perturbation of the Hamiltonian (73) (see [2] for a discussion). If the Hamiltonian (73) (valid only for low l and strictly speaking for

$l = 0$) would be used for calculation of the spectrum for arbitrary values of l, one could obtain [11]

$$M^2 = 2\pi\sigma(2n_r + \frac{\lambda(n_r)}{\pi} l + \delta(n_r, l)) \tag{75}$$

where $\lambda(n_r) \to 4$ when $l \gg n_r$ and $\delta(n_r, l)$ is the small correction for all values of n_r and l. This formula gives a good approximation of the spectrum for not large l. For the Regge trajectory at large l one would get

$$M^2 = 8\sigma l$$

However this approximation gives $\approx 25\%$ deflection from the correct value, as we shall demonstrate below.

Now we obtain the Hamiltonian of the system for arbitrary l. Separating longitudinal and transverse components of $\dot{\vec{r}}$ with respect to $\vec{r}$, we obtain

$$\dot{\vec{r}}^2 = \frac{1}{\vec{r}^2}\{(\dot{\vec{r}}\vec{r})^2 + (\dot{\vec{r}} \times \vec{r})^2\} \tag{76}$$

and the kinetic part of the action (61) can be written as

$$\frac{1}{2}(\frac{\mu(\tau)}{2}\frac{(\dot{\vec{r}}\vec{r})^2}{\vec{r}^2} + (\frac{\mu(\tau)}{2} + \int_0^1 (\beta - 1/2)^2 \nu(\beta, \tau) d\beta) \cdot \frac{(\dot{\vec{r}} \times \vec{r})^2}{\vec{r}^2}) \tag{77}$$

For the longitudinal and transverse components of the momentum one gets respectively

$$p_r^2 \equiv \frac{(\vec{p}\vec{r})^2}{\vec{r}^2} = (\frac{\mu}{2})^2 \frac{(\dot{\vec{r}}\vec{r})^2}{\vec{r}^2} \tag{78}$$

$$p_T^2 \equiv \frac{(\vec{p} \times \vec{r})^2}{\vec{r}^2} = (\frac{\mu}{2} + \int_0^1 (\beta - 1/2)^2 \nu(\beta, \tau) d\beta)^2 \frac{(\dot{\vec{r}} \times \vec{r})^2}{\vec{r}^2} \tag{79}$$

The standard derivation of H from the action in terms of these components yields in the Minkowski space-time

$$H(p, r, \nu, \mu) = \frac{1}{2}(\frac{(p_r^2 + m^2)}{\mu(\tau)/2} + 2\mu(\tau) + \tag{80}$$
$$\frac{\hat{L}^2/\vec{r}^2}{(\frac{\mu}{2} + \int(\beta - 1/2)^2 \nu d\beta)} + \int \frac{\sigma^2 d\beta}{\nu}\vec{r}^2 + \int \nu d\beta)$$

where

$$\hat{L}^2 = (\vec{p} \times \vec{r})^2 = p_T^2 \vec{r}^2 \tag{81}$$

This is the resulting Hamiltonian for the QCD straight-line string with quarks.

Postponing consideration of this general expression to future papers let us now concentrate on the transition of the dynamics from the potential case for small l with the Hamiltonian (73) to the case of large l, which we call the string dynamics.

In the limit $l \to \infty$ one can expand the potential part of the Hamiltonian (80)

$$\frac{1}{2}\left(\frac{\hat{L}^2/\vec{r}^2}{(\mu/2 + \int(\beta - 1/2)^2 \nu d\beta)} + \int \frac{\sigma^2}{\nu} d\beta \vec{r}^2\right) \tag{82}$$

around the extremum in $|\vec{r}|$

$$r_l^2 = (l(l+1)/(\mu/2 + \int(\beta - 1/2)^2 \nu d\beta) \cdot \int \frac{\sigma^2}{\nu} d\beta)^{1/2} \tag{83}$$

Keeping only quadratic terms in $(r - r_l)$ one gets instead of (80)

$$H = 1/2\{\frac{p_r^2 + m^2}{\mu(\tau)/2} + 2\mu(\tau) + \int \nu d\beta + \tag{84}$$

$$2\left(\frac{\sigma^2(l(l+1)) \int \frac{d\beta}{\nu}}{\mu/2 + \int(\beta - 1/2)^2 \nu d\beta}\right)^{1/2} + 4 \int \frac{\sigma^2}{\nu} d\beta (|\vec{r}| - r_l)^2\}$$

Let us show that in the limit $l \to \infty$ the dynamical masses μ and ν satisfy the condition

$$< \mu > \ll < \nu > \tag{85}$$

In this case expanding eq.(84) in μ/ν, we shall obtain the string dynamics regime and the leading Regge trajectory of the Nambu-Goto form

$$M_l^2 \to 2\pi\sigma l \tag{86}$$

Now we prove that the alternative regime

$$< \mu > \ > \ < \nu > \tag{87}$$

yields larger mass values and therefore it is energetically disfavoured. From eq. (84) one can simply estimate that the regime

$$< \mu > \gg < \nu > \tag{88}$$

is not possible for $l \to \infty$ at all and we are left with a possibility

$$< \mu > \approx < \nu > \tag{89}$$

which corresponds to the situation when quarks at the ends carry the fraction of the total energy (orbital momentum) comparable with the string contribution.

In this case one can treat the term $\int (\beta - 1/2)^2 \nu d\beta$ perturbatively. Starting with the value r_l

$$r_l^2 = (\frac{l(l+1)}{\mu} \sigma^2 \int \frac{d\beta}{\nu})^{1/2} \tag{90}$$

and exploiting a numerically small coefficient

$$\int (\beta - 1/2)^2 d\beta = \frac{1}{12}$$

One returns to the Hamiltonian (73) which leads to the trajectory

$$M^2 \to 8\sigma l \tag{91}$$

with a larger mass for a given l than in eq.(86). This demonstrates that the relativistic potential regime is disfavoured, as compared to the "string" one.

We come back now to the string dynamics regime (85) where quarks at the ends carry only small part of the total energy (orbital momentum) of the hadron. In the leading approximation neglecting the dynamics of the longitudinal components one gets

$$S_L^{(0)} = \int_0^T (\frac{1}{2} \int \nu d\beta + (\frac{\sigma^2 l(l+1) \int \frac{d\beta}{\nu}}{\int (\beta - 1/2)^2 \nu d\beta})^{1/2}) d\tau \tag{92}$$

The extremal value of $\nu(\tau, \beta) = \nu_l^{(0)}$ is τ–independent, and we recover the case of the pure string (56)

$$E_l^{(0)} = \frac{1}{2} \int_0^1 \nu(\beta) d\beta + (\frac{\sigma^2 l(l+1) \int \frac{d\beta}{\nu(\beta)}}{\int (\beta - 1/2)^2 \nu(\beta) d\beta})^{1/2} \tag{93}$$

and hence

$$\nu_l^{(0)}(\beta) = (\frac{8\sigma (l(l+1))^{1/2}}{\pi})^{1/2} (1 - 4(\beta - 1/2)^2)^{-1/2} \tag{94}$$

$$(E_l^{(0)})^2 \to 2\pi\sigma l$$

The corrections to the eq.(92) are considered in Appendix C. It should be stressed that while the qualitative dependence of the dynamical mass $<\mu>$ on l is reasonable

$$
\begin{array}{c}
<\mu>\sim l^{\alpha} \quad \alpha > 0 \quad l \to \infty \\
<\mu> / <\nu> \to 0 \qquad l \to \infty
\end{array} \tag{95}
$$

the quantitative results are out of the accuracy of the straight-line approximation.

8 Conclusion

We have applied in this paper the path integral method to quantize the quark-antiquark system interacting nonperturbatively. We have argued that the latter leads to the appearence of a minimal string between the quarks at large distances, $R \gg T_g$, where $T_g \simeq 0.2 fm$ is the correlation length in the vacuum.

As a result our starting Lagrangian consists of kinetic terms for quarks (including the relative time term) and the string part. Using the method of auxiliary fields one gets an effective action quadratic in coordinates and its derivatives. The auxiliary fields are participating in the final action and the integration measure for them is found. We have shown that for the spectrum the integration amounts to taking the local extremum of the action in the values of auxiliary fields.

One of the important problems dealt with in the paper is the question of the relative time. We have shown that this question is resolved for the case of the pure straight-line string due to the reparametrization invariance of the action.

It leads to the constraint $(Pr) = 0$, which corresponds to the condition $R_0 = 0$ in the rest system. We argued that for a string with quarks the same constraint should be used in order to make the straight-line approximation for the surface selfconsistent. It corresponds to the choice of quark trajectories without backward motion in time and defines the valence quark approximation. For this class of trajectories it is possible to eliminate the relative time using the reparametrization invariance of the string term resulting in a Hamiltonian with variable dynamical masses $\mu_i(\tau)$. This leads to the transformation of the initial quadratic kinetic terms to the square - root type ones.

Thus we have obtained the three dimensional dynamics with dynamical variables connected to the dynamical masses of quarks $(\mu_i(\tau))$ and to the effective

dynamical string mass density $\nu(\tau, \beta)$.

The spectrum depends on the relative role of the quark and string degrees of freedom (d.o.f.) which leads to the emergence of two different dynamical regimes. We have obtained it analytically in two limiting cases. (i) States with the total angular momentum $l = 0$. Here in the kinetic part of the action only the quark degrees of freedom contribute while the string provides only the inert part, namely, the exactly linear potential. The Hamiltonian appears to be equal to the so-called "relativistic quark model" [11] one and eigenvalues coincide within the accuracy of our model with those of the proper-time Hamiltonian [2,7] where μ_1 do not depend on τ. This potential-like relativistic regime (when the string carries only a small part of the total orbital momentum l of the hadron) is valid up to moderate values of l smoothly joining the string-like regime at large l. (ii) For the states with $l \gg 1$ the string degrees of freedom dominate. The string carries not only energy, but orbital momentum and cannot be reduced to the potential term only. The effective Hamiltonian for radial degrees of freedom is derived and we show that it yields only small corrections to the string levels.

It is gratifying to note, that the Regge slope of our trajectories is asymptotically $(2\pi\sigma)^{-1}$, the usual string slope, and it differs by $\approx 25\%$ from what one would get from the potential regime (i). The latter slope is $(8\sigma)^{-1}$, the same as was obtained long ago in the so-called relativistic quark potential model [11] and in [2,7]. It should be stressed, that the string slope is energetically favorable as compared to the potential one.

The case of the pure straight-line string (without quarks at its ends) has been treated in detail in Section 6. Results here coincide with those obtained by the canonical quantization method [14]. This case corresponds to the limit (ii) of $l \to \infty$ of the previous one.

One should note also, that our results are in an agreement with numerical quantization of the same quark-string system, done in ref.[21]. There the instantaneous dynamics has been assumed from the beginning and quasiclassical quantization was performed numerically. The authors [21] also have observed two limiting regimes; that of $l = 0$ and $l \gg 1$.

Appendix A

A particle Green's function without backward motion

We shall obtain below the Green's function of a particle in the external field under the condition $\frac{dz_0(\gamma)}{d\gamma} > 0$ for its motion. This condition implies that the only trajectories taken into account are those without additional pair creation, corresponding to the backward in the time motion $\frac{dz_0}{d\gamma} < 0$.

To illustrate the idea we first consider the case of a free particle subject to the same condition

$$\frac{dz_0(\gamma)}{d\gamma} > 0 \tag{A.1}$$

The standard form of the free particle Green's function in the Minkowski space

$$G(x,y) =< x \mid (-\partial^2 + m^2) \mid y >\sim \int \frac{d\tilde{s}}{\tilde{s}^2} Dz_\mu(\gamma) \times \tag{A.2}$$
$$\times \;\; exp[-i \int\limits_0^1 \frac{d\gamma}{2} (\frac{m^2}{\tilde{s}} - \dot{z}_\mu^2 \tilde{s})] = \int_0^{+\infty} d\tilde{s}[exp[\frac{-i}{2}(\frac{m^2}{\tilde{s}} - (x-y)^2\tilde{s})]]$$

contains the summation over all trajectories with any sign of $\frac{dz_0}{d\gamma}$. In the momentum space G has the form

$$G(p_1 \;,\; p_2) \sim \delta^4(p_1 - p_2) \frac{1}{p_1^2 - m^2} \tag{A.3}$$

One can separate in (A.2) the sum over a class of trajectories without backward motion (A.1) as follows. For each trajectory of this class one makes in a unique way the change of integration parameter $d\gamma$ by $\frac{dz_0}{\dot{z}_0}$

$$d\gamma = \frac{dz_0}{\dot{z}_0} \tag{A.4}$$

and the exponent in (A.2) can be rewritten as

$$exp[-i \int\limits_0^1 \frac{d\gamma}{2} (\frac{m^2}{\tilde{s}} - \dot{z}_\mu^2 \tilde{s})] \rightarrow [-i \int\limits_0^T \frac{dz_0}{2} (\frac{m^2}{\tilde{s}\dot{z}_0} + \tilde{s}\dot{z}_0 (1 - \dot{\vec{z}}^2(z_0)))] = \tag{A.5}$$
$$= \;\; exp[-i \int\limits_0^T d\tau (\frac{m^2}{\tilde{s}\dot{z}_0(\tau)} + \tilde{s}\dot{z}_0(\tau)(1 - \dot{\vec{z}}^2(\tau)))]$$

with

$$T = x_0 - y_0 = z_0(T) - z_0(0) \tag{A.6}$$

In the class of trajectories with property (A.1) the transition from the integration over $d\tilde{s}Dz_0$ to $d\mu(\tau)$ with

$$\mu(\tau) = \tilde{s}\dot{z}_0 \tag{A.7}$$

has a nonsingular Jacobean, well known in the string theory [12]

$$D\mu^2(\tau) \sim exp[-i\frac{const}{\epsilon}\int_0^t \sqrt{\mu^2(\tau)}d\tau]d\tilde{s}Dz_0(\tau) \tag{A.8}$$

where $\frac{1}{\epsilon} \sim \Lambda$ is the ultraviolet cut-off parameter. We note that $\tilde{s}$ plays role of a collective coordinate for a set $\{\mu(\tau)\}$, since from (A.7) one has

$$\tilde{s} = \frac{1}{T}\int_0^T d\tau\mu(\tau) \tag{A.9}$$

Combining (A.4-A.8) one obtains for the Green's function $\tilde{G}$ in the class of trajectories (A.1) the following expression

$$\tilde{G} = \int D\vec{z}(\tau)D\mu^2(\tau)exp[-i\int_0^T \frac{d\tau}{2}(\frac{m^2}{\mu(\tau)} + \mu(\tau)(1 - \dot{\vec{z}}^2(\tau)))] \tag{A.10}$$

where a proper rescaling of the mass and $\vec{z}(\tau)$ is made. Note that in this representation the extremum value of $\mu(\tau)$ is

$$\mu(\tau) = m\sqrt{1 - \dot{\vec{z}}^2(\tau)} \tag{A.11}$$

which makes it difficult to treat the case $m = 0$ and trajectories with $\dot{\vec{z}}^2 > 1$.

Hence it is convenient to use the canonical, path integral form where one gets

$$\tilde{G} = \int D\vec{z}(\tau)D\vec{p}(\tau)d\mu(\tau)exp[i\int_0^T (\vec{p}\dot{\vec{z}} - \frac{1}{2}\{\frac{\vec{p}^2 + m^2}{\mu(\tau)} + \mu(\tau)\})d\tau] \tag{A.12}$$

It is important to define the correct integration measure. Having in mind that the following equality holds true

$$\int_0^{+\infty} \frac{dt}{\sqrt{t}} exp[-\frac{a}{2}(t+1/t)] = 2(\frac{\pi}{2a})^{1/2} e^{-a} \tag{A.13}$$

for $\mid \arg a \mid \leq \pi/2$, we are to choose (taking into account eq. (A.8))

$$D\mu(\tau) \sim \sqcap_i \frac{d\mu(\tau_i)}{\mu^{3/2}(\tau_i)}, \tag{A.14}$$

After the integration over $\mu(\tau)$ one obtains for $\tilde{G}$

$$\tilde{G} = \int D\vec{z}(\tau) D\vec{p}\, exp[i \int_0^T d\tau(\vec{p}\dot{\vec{z}} - \sqrt{\vec{p}^2 + m^2})] \tag{A.15}$$

where the exponent is given by the extremum value of $\mu(\tau)$:

$$\mu(\tau) = \sqrt{\vec{p}^2(\tau) + m^2} \tag{A.16}$$

The expression (A.15) is the usual canonical representation for the quantum mechanical Green function with the Hamiltonian

$$H = \sqrt{\vec{p}^2 + m^2}$$

In the momentum representation one has instead of eq.(A.3)

$$\tilde{G}(p_1\ ,\ p_2) \sim \delta^3(\vec{p}_1 - \vec{p}_2)(E_1 - \sqrt{\vec{p}_1^2 + m^2})^{-1} \tag{A.17}$$

Thus the contribution of the trajectories without backward motion is equivalent (for a free particle) to the separation of the positive frequency part from the relativistic propagator.

We take now the case of a particle in the external field A_μ. In an analogous way from the standard form of the Green's function in the external field

$$G(x, y \mid A_\mu) \sim \int \frac{d\tilde{s}}{\tilde{s}^2} Dz_\mu exp[-i \int_0^1 d\gamma(\frac{1}{2}(\frac{m^2}{\tilde{s}} - \tilde{s}\dot{z}_\mu^2) - g\dot{z}_\mu A^\mu)] \tag{A.18}$$

one obtains in the class of trajectories (A.1) the following Green function in external field

$$\tilde{G}_{inv}(x, y \mid A_\mu) \sim \int D\mu(\gamma) Dz_\mu(\gamma) exp[-i \int_0^T d\tau(\frac{1}{2}(\frac{m^2}{\mu(\gamma)} + \mu(\gamma)(1 - \dot{\vec{z}}^2)) - gA_\mu \dot{z}^\mu)] \tag{A.19}$$

In the same way as it has been done for the free particle to transform eq.(A.10) into eq.(A.15) , one can prove that the expression (A.19) leads to usual relativistic Hamiltonian of the particle in external field

$$H(\vec{p}\ ,\vec{x}\ \ A_\mu) = -gA_0(\vec{x},\tau) + \sqrt{(\vec{p} + g\vec{A}(\vec{x},\tau))^2 + m^2} \tag{A.20}$$

Appendix B

The auxiliary field formalism

To develop a procedure of quantization of (29) we use the auxiliary fields formalism, as is usually done in the string theory [3,12].

Let us rewrite (29) as

$$G = \int Dr\ DR\ D\mu\ \ exp[-K' - \bar{K}']\ Dh_{ab} exp[-\sigma_0 \int \sqrt{h} d^2\xi]\ \cdot \tag{B.1}$$

$$\cdot\ \delta(\partial_a w_\mu \partial_b w^\mu - h_{ab}(\xi)) = \int Dr\ DR\ Dh_{ab} \int\limits_{-i\infty}^{+i\infty} D\lambda^{ab} exp[-\sigma_0 \int \sqrt{h} d^2\xi]\ \cdot$$

$$\cdot exp[+\int \sqrt{h}\lambda^{ab} h_{ab} d^2\xi] exp[-\int \sqrt{h}\lambda^{ab}\partial_a w_\mu \partial_b w^\mu d^2\xi] exp[-K - \bar{K}]$$

where

$$d^2\xi = d\gamma d\beta\ ,\quad \xi_1 = \gamma\ ,\quad \xi_2 = \beta\ ,\quad h \equiv deth. \tag{B.2}$$

In the pure string case when the kinetic terms are absent the action (B.1) is invariant under reparametrization $\gamma \to f(\gamma,\beta)$

$$w_\mu(\gamma,\beta) \to w_\mu(f(\gamma,\beta),\beta)\ ,\ h_{11}(\gamma,\beta) \to (\frac{\partial f}{\partial \gamma})^2 h_{11}(f,\beta), \tag{B.3}$$

$$h_{12}(\gamma,\beta) \to (\frac{\partial f}{\partial \gamma}) h_{12}(f,\beta)\ ,\quad h_{22}(\gamma,\beta) \to h_{22}(f,\beta)$$

with the function $f(\gamma,\beta)$ satisfying the conditions

$$f(0,\beta) = 0\ \ , f(1,\beta) = 1\ \ , \frac{\partial f(\gamma,\beta)}{\partial \gamma} > 0 \tag{B.4}$$

It appears to be convenient to decompose [12]

$$\lambda^{ab}(\xi) = \alpha(\xi) h^{ab}(\xi) + f^{ab}(\xi) \tag{B.5}$$

with

$$f^{ab}h_{ab} = 0 \quad , h^{ab} \equiv (h^{-1})^{ab} \tag{B.6}$$

The integral (B.1) takes the form

$$\begin{aligned} G = \int Dr\ DR\ Dh_{ab} exp[-K' - \bar{K}'] \int D\alpha(\xi) Df^{ab}(\xi) \\ exp[-\int (\sigma_0 - 2\alpha(\xi))\sqrt{h} d^2\xi] \cdot \\ \cdot exp[-\int \sqrt{h}((\alpha(\xi)h^{ab} + f^{ab})\partial_a w_\mu \partial_b w^\mu) d^2\xi] \end{aligned} \tag{B.7}$$

We will show now that in the continuum limit $\alpha(\xi)$ and $f^{ab}(\xi)$ can be replaced by their mean values

$$< \alpha(\xi) > \rightarrow \bar{\alpha} \quad , < f^{ab}(\xi) > \rightarrow 0 \tag{B.8}$$

Equation (B.8) reflects the fact, that $\alpha(\xi)$ is a scalar, while $f^{ab}(\xi)$ is a traceless tenzor.

One can write $\alpha(\xi)$ in the form

$$\alpha(\xi) = < \alpha(\xi) > (1 + b(\xi)) \tag{B.9}$$

Since $\alpha(\xi)$ is a scalar, one can expand

$$< \alpha(\xi) > = \bar{\alpha} + c_1 R_1(\xi) + \dots \tag{B.10}$$

where $R_1(\xi)$ is the scalar curvature for the metrics h_{ab}; $\bar{\alpha}$ is some constant with a magnitude much larger (as we will show below) than a characteristic value of $R_1(\xi)$, so that one can neglect all terms except the first one.

To simplify the problem we neglect the dependence of h_{ab} on ξ and put $h_{ab}(\xi) = \delta_{ab}$. In this case it is sufficient to consider a model problem, corresponding to Eq. (B.7)

$$\int D\alpha(\gamma)\ DR(\gamma)\ Dr(\gamma)\ exp(-\int_0^{l} d\gamma \alpha(\gamma)\dot{R}^2) exp(-\int_0^{l} \alpha(\gamma)(\dot{r}^2 + r^2) d\gamma) \tag{B.11}$$

We need the effective action of $\alpha(\gamma)$. The Gaussian integration over $DR(\gamma)$ yields

$$\int \frac{dq}{2\pi} b(q) b(-q) \int \frac{dk}{2\pi} \ \frac{[k(q-k)]^2}{k^2(q-k)^2} \tag{B.12}$$

The integral over dk is linearly divergent and should be cut off at $k \sim \Lambda \to \infty$. Therefore its singularities in the variable q lie at a distance $\sim \Lambda$

$$B(q^2) = \int \frac{dk}{2\pi} \frac{[k(q-k)]^2}{k^2(q-k)^2} \sim \Lambda(1 + const(\frac{q}{\Lambda})^2) \tag{B.13}$$

and the induced action

$$W_R \sim \Lambda \int b^2(\gamma) d\gamma \tag{B.14}$$

suppresses fluctuations of $b(\gamma)$.

The $r(\gamma)$ contribution to the effective action can be obtained with the help of transformation

$$r(\gamma) = f(\gamma) \int\limits_0^\gamma \frac{\dot{z}(\gamma')d\gamma'}{f(\gamma')} \tag{B.15}$$

where $f'' - f = 0$, $f(0) = f(1)$, and we obtain

$$S_r = \int\limits_0^1 d\gamma \alpha(\gamma)(\dot{r}^2 + r^2) = \int\limits_0^1 \alpha(\gamma)\dot{z}^2(\gamma) + \int\limits_0^1 d\gamma \{\alpha(\gamma) \frac{d}{d\gamma}(f'f \int\limits_0^\gamma \frac{\dot{z}}{f} d\gamma')\} \tag{B.16}$$

Note that the Jacobian

$$\frac{Dz}{Dr} = (\frac{f(1)}{f(0)})^{1/2} \tag{B.17}$$

does not depend on $\alpha(\gamma)$.

As in the previous case one obtains a contribution to the effective action from the first term on the r.h.s. of (A.16)

$$W_r \sim \Lambda \int b^2(\gamma) d\gamma \tag{B.18}$$

which damps again fluctuations of $f(\gamma)$, and therefore the second term in expression (A.16) is unimportant.

Thus the field $\alpha(\gamma)$ is connected in fact with two free fields $R(\gamma)$ and $z(\gamma)$. In this case $\bar{\alpha} \sim \Lambda \to \infty$ as shown in [12].

In the same way one can show that the effective action for the fields $f_{ab}(\xi)$ also damps fluctuations of fields

$$W \sim \Lambda \int f^2 d\gamma \tag{B.19}$$

Therefore we have justified the equality (B.8).

After that we obtain the following expression for G:

$$G = \int Dr\ DR\ D\mu_1\ D\mu_2\ Dh_{ab}\ [-K' - \bar{K}'] \cdot \tag{B.20}$$
$$\cdot exp[-(\sigma_0 - 2\bar{\alpha}) \int \sqrt{h} d^2\xi] \cdot exp[-\bar{\alpha} \int \sqrt{h} h^{ab} \partial_a w_\mu \partial_b w^\mu d^2\xi]$$

with the new action which is quadratic in w_μ and contains the new auxiliary fields h_{ab}.

Let us first consider the case of the pure string. The invariance (B.3) makes it convenient to introduce the new variables $\tilde{\nu}(\beta),\ f(\xi),\ \eta(\xi),$ separating out the collective mode $\tilde{\nu}(\beta)$ and the field $f(\gamma, \beta)$, satisfying conditions (B.4)

$$\hbar \equiv \frac{h}{h_{22}^2} = (T\sigma\tilde{\nu}(\beta))^2 (\frac{\partial f(\gamma, \beta)}{\partial \gamma})^2 \tag{B.21}$$

and making a simple rescaling of $\hbar_{12}$.

$$\hbar_{12}(\xi) \equiv \frac{h_{12}}{h_{22}} = (\frac{\partial f(\gamma, \beta)}{\partial \gamma})(T\eta(\gamma, \beta)) \tag{B.22}$$

where T enters the boundary condition. Taking into account the fact, that

$$Dh_{11}\ Dh_{22}\ Dh_{12} = D\hbar\ D\hbar_{12}\ h_{22}^2\ Dh_{22} \tag{B.23}$$

and using the well known formula [12]

$$D\hbar^2 \sim exp[-\frac{const}{\epsilon} \int \sqrt{\hbar} d^2\xi] D\tilde{\nu}(\beta) Df(\gamma, \beta) \tag{B.24}$$

where $1/\epsilon \sim \Lambda$ is the ultraviolet cut-off scale, we arrive at the following expression after change of the integration over $d\gamma$ by $Tdf(\gamma, \beta) \equiv d\tau$

$$G = \int Df Dh_{22} Dr DR D\eta D\tilde{\nu}(\beta) \cdot$$

$$\cdot exp[-(\sigma_0 - 2\bar{\alpha} + \frac{const}{\varepsilon})\sigma \int h_{22}(\xi) \tilde{\nu}(\beta) d\tau d\beta] \cdot$$

$$\cdot\ exp[-\int_0^T d\tau \int_0^1 d\beta \frac{1}{2\tilde{\nu}} ((\frac{\partial w}{\partial \tau})^2 - 2\eta(\frac{\partial w}{\partial \tau} r) + ((\tilde{\nu}\sigma)^2 + \eta^2) r^2)] \tag{B.25}$$

where trivial rescaling $z, \bar{z} \rightarrow (\frac{\sigma}{2\bar{\alpha}})^{1/2} z, (\frac{\sigma}{2\bar{\alpha}})^{1/2} \bar{z}$ together with a proper renormalization of m_0, s in $K, \bar{K}$ has been done.

At first we notice that the action doesn't depend on $f(\gamma,\beta)$ which reflects the invariance (B.23). So that the integral over $Df(\tau,\beta)$ can be factored out and it is equal to the volume of the reparametrization group.

In the standart way [12] we have introduced the physical quantity σ, which entered our expression (B.21)

$$\sigma^2 = \bar{\alpha}(\sigma_0 - 2\bar{\alpha} + \frac{const}{\epsilon}) \tag{B.26}$$

In the general case (B.1) the invariance (B.3) is lost so that we are left with a dependence of $\tilde{\nu}$ on γ, β and have to restrict ourself instead of (B.21), (B.22) by a simple rescaling

$$\hbar \equiv \frac{h}{h_{22}^2} = (T\sigma\tilde{\nu}(\tau,\beta))^2 \tag{B.27}$$

$$\hbar_{12} \equiv \frac{h_{12}}{h_{22}} = (T\eta(\gamma,\beta)) \tag{B.28}$$

After that in the same way we arrive at the expression for G

$$G = \int Dr\ DR\ D\mu\ Dh_{22} D\eta D\tilde{\nu}(\tau,\beta) \tag{B.29}$$

$$\times exp[-K' - \bar{K}'] exp[-(\sigma_0 - 2\bar{\alpha})\sigma \int h_{22}(\xi)\tilde{\nu}(\tau,\beta) d\tau d\beta]$$

$$\times exp[-\int_0^T d\tau \int_0^1 d\beta \frac{1}{2\tilde{\nu}}((\frac{\partial w}{\partial \tau})^2 - 2\eta(\frac{\partial w}{\partial \tau} r) + ((\tilde{\nu}\sigma)^2 + \eta^2) r^2)]$$

which differs from (B.45) by three points; (i) changing factor $(\sigma_0 - 2\bar{\alpha} + \frac{const}{\epsilon})$ by $(\sigma_0 - 2\bar{\alpha})$ (ii) an explicit dependence of $\tilde{\nu}(\tau,\beta)$ on τ (iii) by the presence of kinetic terms.

Finally after gaussian integration over $h_{22} \geq 0$ we obtain for the case of the pure string

$$G = \int DR_\mu\ Dr_\mu\ D\tilde{\nu}(\tau) d\eta(\tau,\beta) exp[-A_{str}] \tag{B.30}$$

where

$$A_{str} = \int_0^T d\tau \int_0^1 d\beta \frac{1}{2\tilde{\nu}} [\dot{w}^2 + (\sigma\tilde{\nu})^2 r^2 - 2\eta(\dot{w}r) + \eta^2 r^2] \tag{B.31}$$

and in the general case (B.1)

$$G = \int DR_\mu\ Dr_\mu\ D\mu\ D\tilde{\nu}(\tau,\beta) d\eta(\tau,\beta) exp[-A_{str} - K' - \bar{K}'] \tag{B.32}$$

Appendix C

In this Appendix we derive the effective Hamiltonian for the longitudinal excitations, calculate the corrections to the string result (85), and consider the behaviour of the dynamical mass μ as a function of l.

It is easy to prove that after taking into account the longitudinal dynamics one obtains a small correction to (93) of the order of

$$\mu^{-1/2} l^{-3/4}$$

Since μ is weakly growing with l, as we shall see below, one concludes that at $l \to \infty$ the field $\nu(\tau, \beta)$ separates and is governed by its own purely string dynamics, only weakly perturbed by the dynamics of $r^2(\tau)$ and $\mu(\tau)$. The latter are "living" in the external field $\nu(\beta)$.

We now compute the longitudinal contribution to the energy of the system to the leading order in $(1/l)$. To this end one can use the nonperturbed value $\nu_l^{(0)}$ (93) and make an expansion in (83) as follows.

$$2\left(\frac{\sigma^2 l(l+1)\int \frac{d\beta}{\nu^{(0)}}}{\frac{\mu}{2} + \int(\beta-1/2)^2\nu^{(0)}d\beta}\right)^{1/2} = 2\left(\frac{\sigma^2 l(l+1)\int \frac{d\beta}{\nu^{(0)}}}{\int(\beta-1/2)^2\nu^{(0)}d\beta}\right)^{1/2} + \tag{C.1}$$
$$+(-1)\frac{\mu}{2}\frac{(\sigma^2 l(l+1)\int \frac{d\beta}{\nu^{(0)}})^{1/2}}{(\int(\beta-1/2)^2\nu^{(0)}d\beta)^{3/2}} + 3/4\left(\frac{\mu}{2}\right)^2\frac{(\sigma^2 l(l+1)\int \frac{d\beta}{\nu^{(0)}})^{1/2}}{(\int(\beta-1/2)^2\nu^{(0)}d\beta)^{5/2}}$$

Insertion of $\nu^{(0)}(\beta)$ from (93) into (C.1) yields

$$(2\pi\sigma(l(l+1))^{1/2})^{1/2} - 2\mu + \frac{3\cdot\sqrt{2}}{\sqrt{\pi}}(\mu)^2(\sigma(l(l+1))^{1/2})^{-1/2} \tag{C.2}$$

and finally one gets

$$H = \frac{1}{2}\left[\frac{p_r^2+m^2}{\mu/2} + 2(2\pi\sigma)^{1/2}(l(l+1))^{1/4} + \right. \tag{C.3}$$
$$\left. + \frac{\mu^2}{4}\frac{12\sqrt{2}}{\sqrt{\pi}(\sigma^2 l(l+1))^{1/4}} + \frac{\sigma^{3/2}\pi^{3/2}(r-r_0)^2}{2^{3/2}(l(l+1))^{1/4}}\right] =$$
$$= E_L^{(0)} + H^{(r)}$$

where $H^{(r)}$ is an effective Hamiltonian for the radial excitations of the hadron

After insertion in this expression of the extremal value of μ

$$\mu = (\frac{p_r^2+m^2}{3})^{1/3}(\frac{\pi\sigma l}{2})^{1/6} \tag{C.4}$$

we finally obtain for the effective radial Hamiltonial

$$H^{(r)} = \frac{1}{2}(3^{4/3}(\frac{2}{\pi\sigma l})^{1/6}(p_r^2+m^2)^{2/3} + (\frac{\pi\sigma}{2})^{3/2}l^{-1/2}(r-r_0)^2) \tag{C.5}$$

where the substitution $(l(l+1))^{1/2} \to l$ has been made in the limit $l \to \infty$.

Let us consider the case of massless current quarks

$$m = 0 \tag{C.6}$$

Introducing instead of $(r-r_0)$ a new dimensionless variable x

$$(r-r_0) = 3^{2/5}l^{1/10}(\frac{\pi\sigma}{2})^{1/2}x \tag{C.7}$$

we represent eigenvalues of the Hamiltonian (C.5) in the following way

$$E^{(r)}_{l,n_r} = l^{-3/10}(\frac{\pi\sigma}{2})^{1/2}3^{4/5}a(n_r) \tag{C.8}$$

where $a(n_r)$ is the eigenvalues of the new dimensionless Hamiltonian

$$\tilde{H}^{(r)} = \frac{1}{2}((-\frac{d^2}{dx^2})^{2/3} + x^2) \tag{C.9}$$

In order to obtain the approximate value $a(n_r)$ we consider eq. (C.3) for the restricted class of functions μ independent on τ. Such procedure in general gives accuracy about 5% for low lying states. In this way one can easily get

$$a(n_r) = 2^{-1/5}\cdot 3^{-3/5}\frac{5}{4}(n_r+\frac{1}{2})^{4/5} \tag{C.10}$$

Substituting this expression into eq. (C.8) we arrive at the final expression for the total energy E_{l,n_r} of the hadron and have for the mass squared

$$M^2_{l,n_r} = 2\pi\sigma(l + const\, l^{1/5}(n_r+1/2)^{4/5}) \tag{C.11}$$

which is slightly different in the case $l \to \infty$ from the pure string result (85).

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